An Evaluation Method of Probability of Elastic-Plastic Fracture by 2-Parameter Criterion

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Many researchers have made a lot of progress in studying the evaluation of fracture probability of brittle materials. However, studies of fracture probability for elastic-plasticity have not been made yet. An evaluation method for fracture probability which is grafted onto a 2-parameter criterion and statistical probability analysis is not only introduced in this study, but also applied to the simple 2-dimensional model and carbon steel piping to vealuate the effect of statistical variables.

Key Words : Fracture Probability, Elastic-Plastic Fracture, 2-Parameter Criterion, Failure Assessmet Diagram, Carbon Steel Pipes

1. Introduction

In spite of technological improvements in industrial society, there has been an increasing number of work-related accidents on many industrial sites. Recently, industrial structures and machinery have become excessively large and complex due to increased business competitions. This has resulted in more accidents and an increasing loss of lives and economic damage. In general, the damage and fractures of structural components occurs in the most susceptible areas. Also, a defective crack that provided during the manufacturing process or the operation of the structure grows and results in a fracture. In this case, the safety of structural components is influenced by the size of the defects and micro structure of materials. The reliability and safety based on the statistical probability evaluation method for the damage should be evaluated since the defects and micro formations show the probability distribution characteristics.

The research grafting fracture mechanics and reliability engineering to predict the fracture probability has been pursued by many researchers (Bloom, 1984 ; Provan, 1987 ; Virkler et al., 1979 ; Kitagawa et al., 1986 ; Okamura et al., 1975 ; Haris, 1977 ; Besuner and Tetelman, 1977 ; Okamura and Itagaki, 1983 ; Yoon and Okamura, 1989 ; Yoon, 1990). The results are used for evaluating safety and reliability. However, it is

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hard to apply this method to an elastic-plastic fracture accompanied by a large plastic deformation. It has to be based on the elastic-plastic fracture mechanics.

The fracture theories based on elastic-plastic mechanics are the J-integral method and the 2-Parameter Criterion. The 2-Parameter Criterion is applied to the real structure because of its comparatively preferable applications.

The 2-Parameter Criterion is based on the Dugdale model (Dugdale, 1960) on the assumption that it is perfect elastic plasticity, which was developed by Central Electricity Generating Board (CEGB) and called R6 (Milne, 1983). Numerous modifications for the 2-Parameter Criterion have been pursued through application and evaluation on real instruments and structures (Shih et al., 1983 ; Bloom, 1983 ; Kobayashi et al., 1987).

However, it is rare to find an evaluation method for structural fracture probability used by statistical and stochastic methods based on elastic-plastic mechanics. In this study, an evaluation method for structural fracture probability on a structural sub-material with a defect is presented by a 2-Parameter Criterion based on elastic-plastic mechanics. This method will be applied to the general machinery structures and fracture probability will be determined. Finally, the effects of parameters to estimate the fracture probability will be discussed.

2. The Development of Simulation Program

In this study, The computer simulation method of predicting the fracture probability for elastic-plastic materials which were discontented with small scale yielding has been established, Fracture probability was obtained from particular structure materials, while several effects of parameters were discussed. The flowchart of program is presented in Fig. 1.

As a method of study, the fracture assessment in structural materials was based on the 2-Parameter Criterion so that a computer program could be developed. Statistical method, Monte Carlo

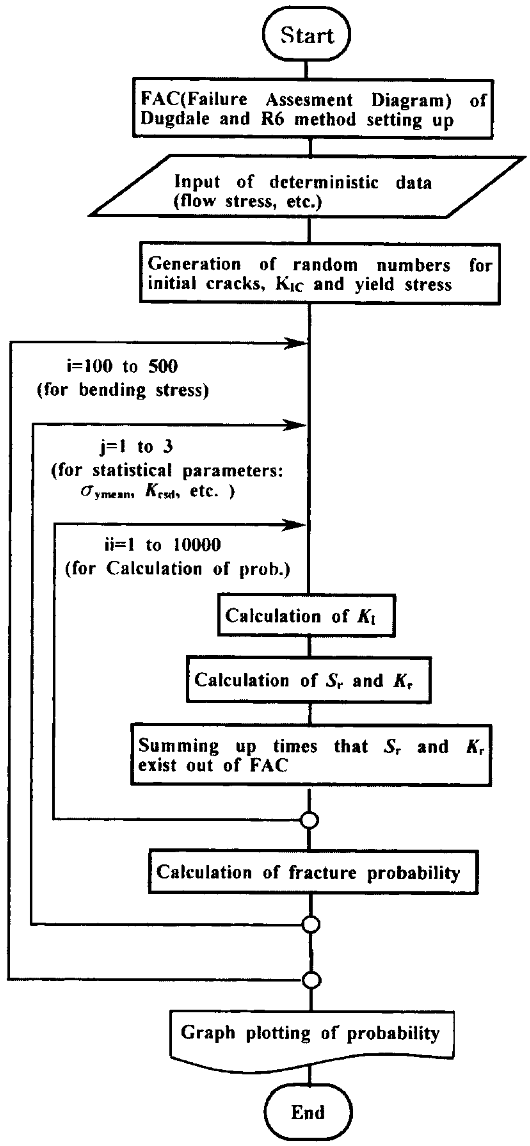

Fig. 1 Flowchart of computer program

Simulation ; probability calculation graph, and other programs were developed. Then, these two programs were fused.

3. Fracture Assessment by 2-Parameter Criterion

3.1 Failure assessment curve (FAC)

The 2-Parameter criterion was developed by using the Dugdale model. The crack opening displacement of the crack tip in a center crack in an infinite plate subjected to a remote tensile stress is given by the following relationship (Dugdale, 1960).

$$\delta=\frac{8\sigma_y a}{\pi A}\ln\left[\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_y}\right)\right] \quad (1)$$

where, $2a$ is crack length, σ is applying stress, σ_y is yield strength, E is Young's modulus. Regardless of yield scale, assuming that failure occurs when it is $\delta=\delta_c$ (critical crack opening displacement) in accordance with crack opening displacement condition. Fracture toughness K_c as the following equation :

$$\delta_c=\frac{K_c^2}{E\sigma_y} \quad (2)$$

Therefore, from Eq. (1) and Eq. (2) stress σ of any yield scale is determined. Also, stress intensity factor K, dealing with σ, is defined by the following equation.

$$K=\sigma\sqrt{\pi a} \quad (3)$$

make K, σ, non-dimension and then define K_r, S_r

$$K_r=\frac{K}{K_c} \qquad S_r=\frac{\sigma}{\sigma_y} \quad (4)$$

Also, using Eq. (1) and (4), the relation between K_r and S_r is obtained by

$$K_r=\left[\frac{8}{\pi^2 S_r^2}\ln\left\{\sec\left(\frac{\pi}{2}S_r\right)\right\}\right]^{-\frac{1}{2}} \quad (5)$$

Through Eq. (5), when failure occurs the locus of K_r and S_r can be calculated, and this locus is defined as the Failure Assessment Curve (FAC). Figure 2 shows FAC which takes K_r for vertical axis and S_r for horizontal axis. It is defined as the Failure Assessment Diagram (FAD).

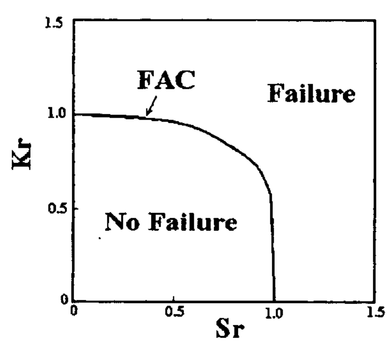

Fig. 2 Failure assessment diagram using Dugdale model

In the small scale yielding, fracture occurred with $K_r=1$ at vertical axis ; and in the full scale yielding, plastic collapse comes from $S_r=1$ at horizontal axis. In addition, fracture occurred on the FAC corresponding to point for a certain scale yielding.

The R6 method based on the Dugdale model was revised and the revision 3 of R6 has already been published. Revision 3 contains three types of options.

Option 1 : It is adapted to many materials. The stress-strain curve of materials is continuous.

$$K_r=(1-0.14L_r^2)\left[0.3+0.7\exp(-0.65L_r^6)\right] \quad (6)$$

Option 2 : It is adapted to materials when the rate of primary strain hardening is high and the stress-strain curve is discontinuous.

$$K_r=\left(\frac{E\varepsilon_t}{\sigma_t}+\frac{\sigma_y L_r^3}{2E\varepsilon_t}\right)^{-1/2} \quad (7)$$

where, E is the elastic modulus and ε_t is the true strain.

Option 3 : It is adapted in case that the FAC of peculiar materials and shapes of the structural components can be obtained by J-integral analysis.

$$K_r=\left(\frac{J_e}{J}\right)^{1/2} \quad (8)$$

where, J_e is the elastic property of J-integral, J. L_r used in the above equations is

$$L_r=\frac{P}{P_y} \quad (9)$$

where, P is load and P_y is plastic collapse load (the yield strength σ_y). L_r is the same as S_r under tensile loading and is restricted by following $L_r^{\max}$.

$$L_r^{\max}=\frac{P_f}{P_y} \quad (10)$$

where, P_f is plastic collapse load under consideration of strain hardening. σ_y of P_y is replaced with flow stress σ_f (the mean value of yield strength σ_y and tensile strength σ_B).

Option 1 was applied in this study because of its simple application. However, in the other cases, it is possible to apply other options through minor modification.

3.2 Deterministic evaluation of FAD by the simple 2-dimensional model

Evaluate the FAD by the simple 2-dimensional model in Fig. 3. In this case, stress intensity factor (Anderson, 1995) is

$$K_I = \sigma\sqrt{\pi a}F(\xi) \tag{11}$$

where, $\xi = a/W$, $F(\xi)$ is,

$$F(\xi) = \sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}}\,\frac{0.923+0.199\{1-\sin(\pi\xi/2)\}^4}{\cos(\pi\xi/2)} \tag{12}$$

The carbon steel STS42 which is used in the generating plant is considered as an applicable material. Material properties are shown in Table 1.

In the calculation, $a=10\sim30$[mm], $W=10$ [cm] and $\sigma=100\sim300$[MPa] is adapted to Eq. (11) and (12). K_c in Eq. (4) is gained by

Table 1 Material properties of STS 42

Parameter	Value
Yield strength σ_y	362[MPa]
Tensile strength σ_B	592[MPa]
Flow stress σ_f	477[MPa]
Elastic modulus E	178.4[GPa]
Fracture toughness J_{Ic}	152[kJ/m^2]

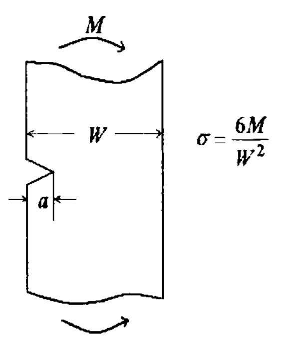

Fig. 3 The 2-dimensional model for evaluation of FAC

$$K_{Ic}(J) = \left(\frac{J_{Ic}\cdot E}{1-\nu^2}\right)^{\frac{1}{2}} = \left(\frac{152[\mathrm{kJ/m^2}]\cdot 178.4[\mathrm{GPa}]}{1-0.3^2}\right)^{\frac{1}{2}} = 172.62[\mathrm{MPa}\sqrt{m}] \tag{13}$$

And, since L_r^{max} as the critical value is related to yield strength $\sigma_y \fallingdotseq 362$[MPa], tensile strength $\sigma_B \fallingdotseq 592$[MPa] and flow stress $\sigma_f = \frac{\sigma_y+\sigma_B}{2} = 477$ [MPa], therefore,

$$L_r^{max} = \frac{P_f}{P_y} = \frac{\sigma_f}{\sigma_y} = 1.318 \tag{14}$$

The results calculated by applying these equations are plotted in Fig. 4. The thick solid lines are FAC which represent the Dugdale model and R6-rev.3. The thin solid line shows the calculation results of Eq. (6) and (9): its lengths of initial crack are 10[mm], 20[mm] and 30[mm], while the bending stress has changed 100[MPa] to 300[MPa]. The Figure shows that it is safe in the case of the small bending stress since the thin solid line exists inside the FAC, but it's not safe as the bending stress becomes larger since the line exists outside the FAC. There is no practical difference between the Dugdale model and R6-rev.3 when the crack length is large. However, when the crack length is small and the bending stress is large (part of circle), the Dugdale model and R6-rev.3 are different. For instance, as shown in the line (a=10[mm]), it fails according to the Dugdale model, but is safe according to R6-rev.3.

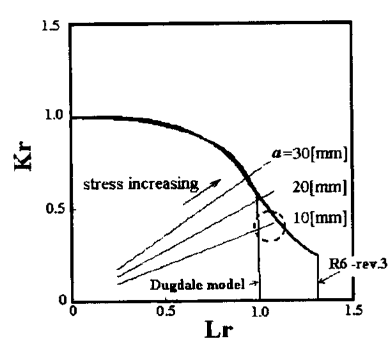

Fig. 4 Failure evaluation in FAD

4. Probabilistic Evaluation by 2-Parameter Criterion

In the statistical method, fracture probability is determined by a simple calculation from probability distribution, numerical analysis and the use of the Monte Carlo Simulation. When probability distribution models of the parameters which reign the failure are the same, computation is possible by simple calculation or numerical integration. But, when the distribution models are different, it must be calculated by using the Monte Carlo Simulation. In this study, fracture probability is calculated by using the Monte Carlo Simulation.

The fracture probability is obtained as the rate of times that K_r and S_r induced from such probability functions as the initial crack length, the fracture toughness and the yield strength exist outside FAC for the total number of trials (10,000 times). Eq. (15) is the fracture probability formula.

$$\begin{aligned} P_f &= \int_{K_r=1} f(K_r) \quad (S_r,\ L_r=0) \\ &= \int_{K_r(S_r)} f(K_r) \quad (S_r,\ L_r>0 \text{ and } K_r>0) \quad (15) \\ &= \int_{S_r=1, L_r^{max}} \quad (K_r=0) \end{aligned}$$

4.1 Probability evaluation by 2-dimension model

In the calculation of the fracture probability, the length of the initial crack, the yield strength and the fracture toughness are treated as the probabilistic variables. There are some kinds of distribution information of the initial crack size, such as exponential distribution (Jouris and Shaffer, 1980), log-normal distribution (Nilsson, 1979) and Gamma distribution (Nilsson, 1979). In this paper, we assume that the initial crack length distribution follows the exponential distribution.

Normal distribution and log-normal distribution were reported to the distribution of yield strength or fracture toughness (Okamura and Itagaki, 1983). In this study, normal distribution was used.

Table 2 Input data for calculation

Parameter	Value
Width [mm]	100
Stress [MPa]	100~500
Non-dimensional initial crack length (Exponential distribution : λ)	5, 10, 15
Yield strength [MPa] (Normal distribution : mean σ_{ymean})	300, 350, 400
Yield strength [MPa] (Normal distribution : Standard dev. σ_{ysd})	5, 20, 35
Fracture toughness [MPa$\sqrt{m}$] (Normal distribution : mean K_{cmean})	120, 170, 230
Fracture toughness [MPa$\sqrt{m}$] (Normal distribution : Standard dev. K_{csd})	5, 10, 15

The initial crack length is used as the non-dimensional variables when divided by width W. Input data for calculation is shown in Table 2. In the table, λ is the exponential distribution parameter of the initial crack length, σ_{ymean} and σ_{ysd} are the mean and standard deviation of the yield strength, and K_{cmean} and K_{csd} are the mean and standard deviation of the fracture toughness. Mean and standard deviation of the non-dimensional initial crack length is 0.2, 0.1, and 0.067, because the relationship between the parameter of exponential distribution λ and mean and standard deviation of normal distribution is reciprocal.

4.1.1 The effect of initial crack variation

Fracture probability is obtained by changing the distribution parameter λ to 5, 10, 15 as the non-dimensional initial crack length follows the exponential distribution, as shown in Fig. 5. Another probability variables are given as the middle values (ex. mean is 350[MPa], standard deviation is 20[MPa] in the case of yield strength) were applied. In the Figure, the vertical axis is cumulative probability of fracture and the horizontal axis is the bending stress. This Figure falls on the normal probability paper, because the scale of vertical axis is the normal distribution probability. The oblique line is subject to the Dugdale model and the solid line is subject to

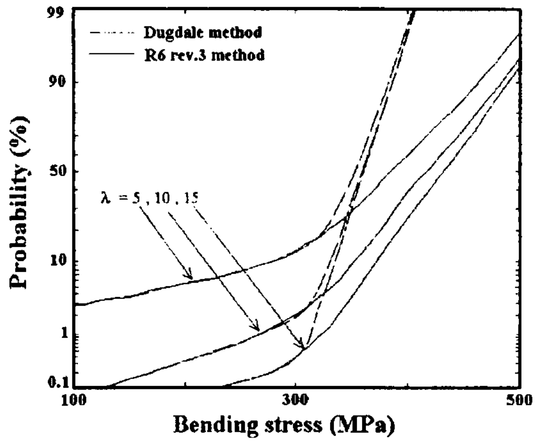

Fig. 5 Effect of initial crack size distribution

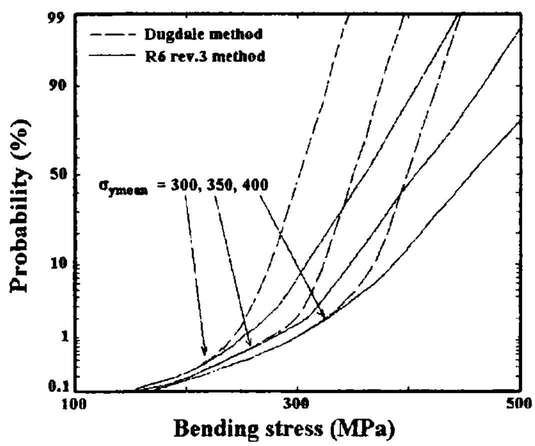

Fig. 6 Effect of mean yield strength

R6-rev.3. In Fig. 5, There is no practical difference between the two models when the bending stress is small. Yet, the difference increases as the bending stress increases. Also, the effect of initial crack distribution is definite when the bending stress is small. The effect decreases as the stress increases.

4.1.2 The effect of yield strength variations

It is assumed that the distribution of yield strength follows the normal distribution. The standard deviation of yield strength was fixed 20 [MPa], and the mean was changed at 300[MPa], 350[MPa] and 400[MPa]. The distribution parameter of the initial crack length, the mean and

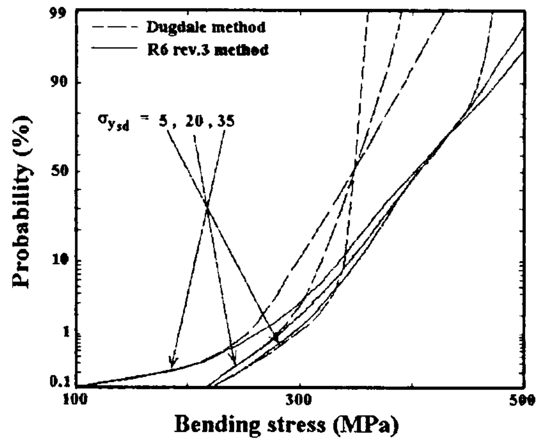

Fig. 7 Effect of standard deviation of yield strength

standard deviation of fracture toughness were fixed on the middle value. The results are shown in Fig. 6. From Fig. 6, it was found that the effect of the mean yield strength becomes considerably smaller as the bending stress decreases, but the effect becomes larger when the stress increases.

The results that the mean yield strength was fixed at the middle value (350[MPa]). The standard deviation was changed at 5[MPa], 20[MPa] and 35[MPa] are shown in Fig. 7. In this case, it was shown the complicated appearances that the effect of the standard deviation of yield strength increases gradually and it is eventually reversed.

4.1.3 The effect of fracture toughness variation

Figure 8 represents the effect of the mean fracture toughness. The mean fracture toughness was changed at 120 MPa$\sqrt{m}$, 170 MPa$\sqrt{m}$ and 230 MPa$\sqrt{m}$. The other distribution parameters were fixed at a middle value, same as before. As shown in Fig. 8, the effect of the mean fracture toughness is always large when the bending stress is small. But as the bending stress increases, the effect in the Dugdale model decreases rapidly, while the effect in R6 rev.3. decreases more slowly.

The effect of the standard deviation of fracture toughness which was changed at 5 MPa$\sqrt{m}$, 15 MPa$\sqrt{m}$, and 25 MPa$\sqrt{m}$, are shown in Fig. 9. As shown in the figure, the effect is very small,

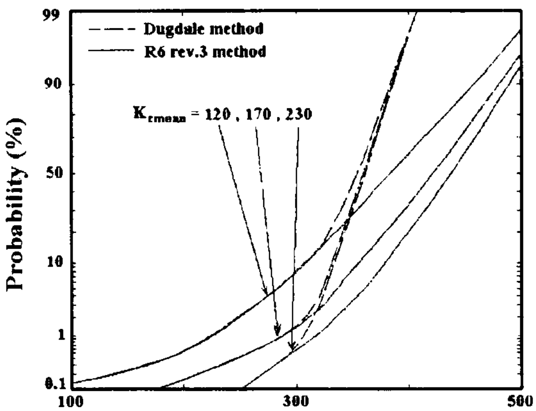

Fig. 8 Effect of mean fracture toughness

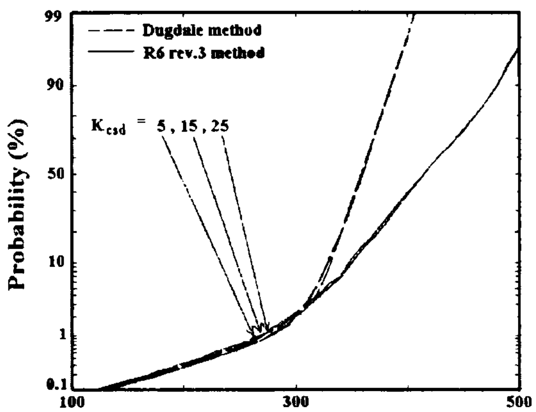

Fig. 9 Effect of standard deviation of fracture toughness

because the shapes of curves are almost the same in spite of the difference in standard deviation of fracture toughness.

4.2 Application of the carbon steel pipe in the power plant

In this paragraph, The effect of several parameters is discussed by applying the evaluation method of the fracture probability to the real structure. It is assumed that the bending moment is loaded on the carbon steel pipe with circumferential crack in Fig. 10.

As Fig. 10 illustrates, when the bending moment M is loaded on the steel pipe, the bending stress is as follows

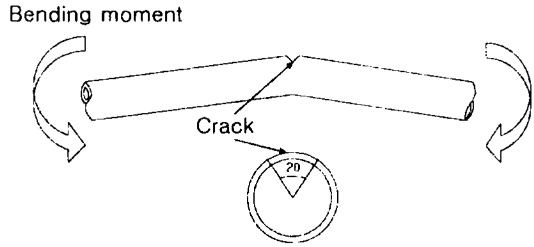

Fig. 10 Condition of loading and shape of crack

$$\sigma_b = \frac{M}{2\pi R^2 t} \tag{16}$$

where t is thickness of pipe, R is average radius. The stress intensity factor (Tada et al., 1985) of the crack tip is given by

$$K = \sigma_b \sqrt{\pi (R\theta)} \cdot F(\theta) \tag{17}$$

where, θ is the half angle of the crack, $F(\theta)$ is the correction factor:

$$F(\theta) = 1 + 6.8\left(\frac{\theta}{\pi}\right)^{\frac{3}{2}} - 13.6\left(\frac{\theta}{\pi}\right)^{\frac{5}{2}} + 20\left(\frac{\theta}{\pi}\right)^{\frac{7}{2}} \tag{18}$$

In the calculation of fracture probability, the size of the initial crack, the yield strength and the fracture toughness were referred to as the probability variables as in the 2-dimensional model. The half angle (θ) of the crack for the size of crack was considered as the variable.

With the input data, the size of the crack is different from the 2-dimensional model, but the others are the same. The value of exponential distribution parameter λ on the half angle (θ) of the crack is 1/5, 1/10, 1/15. It is related to 5°, 10°, 15° in the mean and standard deviation of normal distribution. In this paragraph, we only consider R6 rev.3 as the criterion of failure.

4.2.1 The effect of the variation of the initial crack angle

The results that are calculated after changing the exponential distribution parameter of initial crack angle to 1/5, 1/10, 1/15 are shown in Fig. 11. The effect is large when the bending moment is small, and the effect decreases as the bending moment becomes larger like the 2-dimensional model.

For the 2-dimension model, curves change

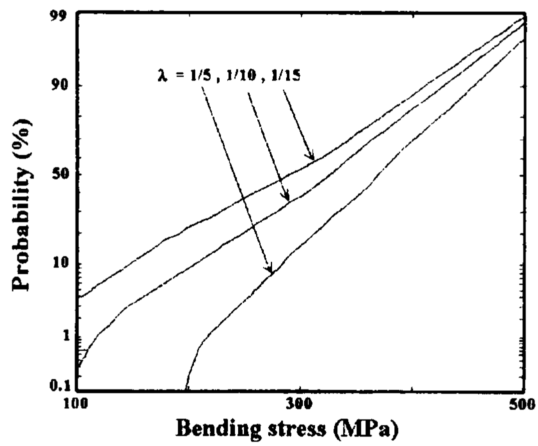

Fig. 11 Effect of crack size distribution in carbon steel pipes

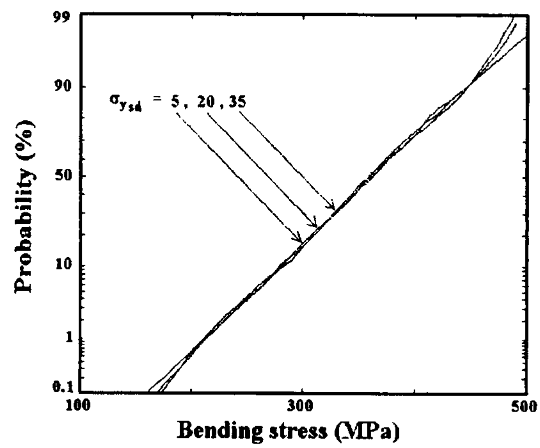

Fig. 13 Effect of standard deviation of yield strength in carbon steel pipes

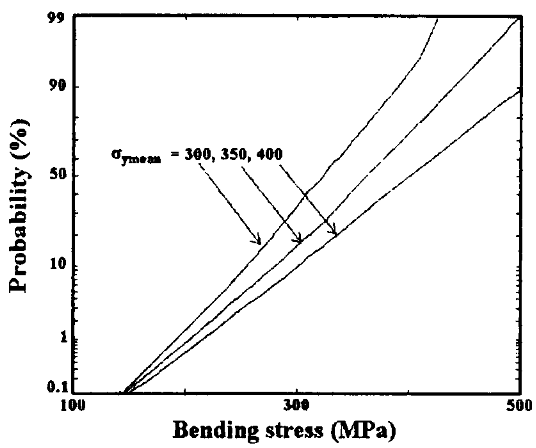

Fig. 12 Effect of mean yield strength in carbon steel pipes

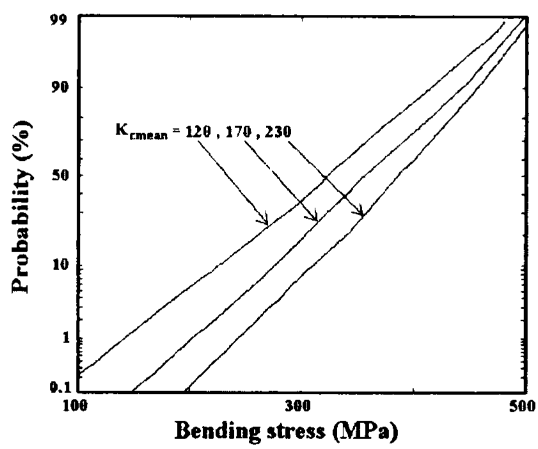

Fig. 14 Effect of mean fracture toughness in carbon steel pipes

slowly at small stress, but change abruptly at large stress. However, for carbon steel pipes, the curves change abruptly at small stress and change slowly as stress becomes larger.

4.2.2 The effect of variation of the yield strength

Figure 12 shows the effect of the mean yield strength on fracture probability. The mean yield strength was changed at 300, 350 and 400[MPa] as same as in the 2-dimensional model. The effect is similar to the 2-dimensional model.

Figure 13 shows the difference in the fracture probability by changing the standard deviation of yield strength. The standard deviation of yield strength was changed at 5, 20 and 35[MPa]. As shown in Fig. 13, there is not nearly the effect on the standard deviation. The result is different from the 2-dimensional model's.

4.2.3 The effect of the variation of the fracture toughness

The results of calculation of fracture probability for the mean and standard deviation of fracture toughness as variable are shown in Figs. 14 and 15. The effects are similar to those in the 2-dimension model except for some differences in the small bending stress.

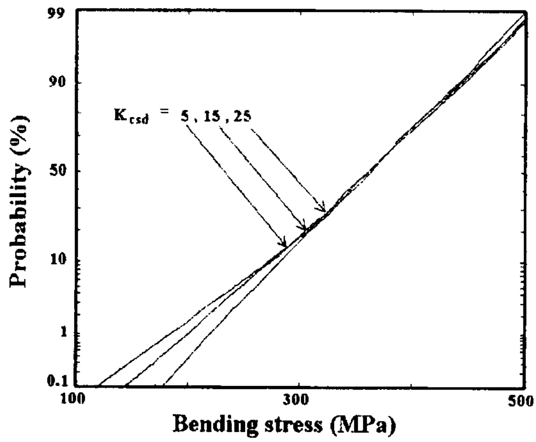

Fig. 15 Effect of standard deviation of fracture toughness in carbon steel pipes

5. Conclusions

In this study, a computer program was developed that predicts fracture probability for the elastic-plastic materials by the 2-parameter criterion. The effects of the distribution parameters on fracture probability were discussed by adapting the simple 2-dimensional model to the program. Also, the effect of parameters were discussed by applying the method of the fracture probability to the real structure.

The results obtained from this study are as follows :

(1) The result from the deterministic evaluation of FAD is that it is safe in the case of the small bending stress since the thin solid line exists inside the FAC, but it's not safe when the bending stress becomes larger since the line exists outside the FAC.

There is no practical difference between the Dugdale model and R6-rev.3 when the crack is large. However, when the crack is small, the Dugdale model and R6-rev.3 are different.

(2) The result from the probability evaluation by the 2-dimensional model is that the effect of mean yield strength becomes considerably smaller as the bending stress decreases, but the effect is larger when the stress increases. For the standard deviation of yield strength, the effect increases gradually and it is eventually reversed. The effect of the mean fracture toughness always is large when the bending stress is small. As the bending stress increases, the effect in the Dugdale model decreases rapidly, while the effect in R6 rev.3. decreases more slowly.

(3) The result from applying the evaluation method of fracture probability to the real structure is that the effect is large when the bending moment is small, and the effect decreases as the bending moment becomes large like the 2-dimensional model.

For the 2-dimensional model, curves change slowly at small stress, and change abruptly at large stress. However, for carbon steel pipes, the curves change abruptly at small stress and change slowly as stress becomes larger. In the case of the effect on the fracture probability by changing the standard deviation of yield strength, there is not nearly the effect of the standard deviation. The result is different from the 2-dimensional model's.

The effects of the variation of the fracture toughness are similar to those in the 2-dimensional model except for some differences in the small bending stress.

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